THE MODELLING OF ENVIRONMENTAL-ECONOMIC SYSTEMS

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Abstract: Any national economy develops in a complex network of intersectoral linkages. In the today's world the specialist from the different fields of knowledge constantly have to face the challenges created by the complexity of the world itself of both natural (nature) and man-created (technosphere) causes. To solve this problem successfully it's not enough to consider individual elements or particular questions separately. They have to be considered as a whole, in a system, taking into account many interconnections and specific traits. For example, the demand for cars influences not only car industry but also indirectly influences the steel industry as its product is central for car production and the industries related to the manufacturing of tires and other components and also the industries producing car conditioners, radios, computers. The method of interindustry analysis also known as input-output analysis developed by the American economist V.V. Leontiev allows to give a coherent and numerical answer to the questions connected to interindustrial impact and its influence on main macroeconomic indicators. The article presents the main challenges connected with the modelling of environmental-economic system and also model examples calculations and the analysis of the results of the modelling.

Keywords: Ecology, Modeling of Ecological and Economic Systems, Nature, Economics of Nature Management, Technosphere.

1. INTRODUCTION

The connection between the social and the economic was first considered in the frame of the classical political economy by Quesnay – Adam Smith – Ricardo. But in the end of the XIX century after «Walras's revolution» and the advent of «neoclassical» political economy the concept of «pure economy» as a science appeared. The main focus was on the questions of the market, the price's evolution, the movement of capital, etc. They still attract scientists' attention and these problems establish the foundation for the development of mathematical models and new mathematical methods. Besides there already exists a diverse effective toolbox to solve many important problems. In the end of the 70-s of the XX century it became obvious that these approaches cannot provide quantitative analysis of the economic development perspectives and the assessment of the variants of the targeted international actions and also the solution of the complex problem of human-environment interaction. It was Dr. Forrester who introduced new approaches; in his fundamental work "World Dynamics" the processes of the economic development, demographics and environmental pollution were «gathered together» in one mathematical model to describe a global environmental process. This fact entails a conceptually-new approach that allows the transition from a concept of economic system to a concept of environmental-economic system. Naturally there appear new specific goals and models [6; 7; 11; 12; 14].

2. RESULTS

2.1. THE PECULIARITIES OF ENVIRONMENTAL-ECONOMIC SYSTEM

Environmental-economic systems are very structurally complex system as they encompass many different objects and fields. The management of these systems is based on the control theory. We will use the term «control» to refer to the process of the influence on a system or an object (controlled object) which changes the state of the system or the object in «a desired way». Obviously controlled objects include mechanisms (for example, a car), economic situation of a factory or a firm, a region's ecosystem, a process of project development, a software project itself and its characteristics, etc. A controlled object is considered to be a system of any degree of complexity that transforms control input U(t) into output signals (trajectories) V(t) that indicate the state of the controlled object in a point in time t. Obviously a real-life controlled object can have a lot of input and output points that define its functional interactions with the environment.

A controlled object and a control device that influences it form together a control system. A controlled object is also supposed to be influenced by interferences that change the main characteristics of a controlled object unpredictably as a rule. An example of a controlled object can be a factory that processes raw materials and outputs finished goods. Here 'control' refers to the whole system of activities and innovations aimed for reaching a goal or goals connected, for example, with the quality and quantity of the output product and also with requirements for the launch date. The description of an object as a controlled object and the identification of the impact pathways can be made only taking into account the intended control target. For example, from the point of view of the sanitary and epidemiological inspection or other environmental services a facility is a controlled object which process raw materials into factory waste which pollutes the environment. In this case the control issued by these services will be aimed to the reduction of the influence of harmful industrial factors and not to the production intensification [4; 10]. The development of economy influences the development and the state of social and environmental areas greatly. The nature and the direction of the impact defines the qualitative side of the economic development [4; 5; 6; 7; 9].

At the same time the economical development mainly depends on the natural resources available. The availability, the quantity and the quality of natural resources in a territory, determines their customer and production uses. Both anthropogenous and natural factors lead to the environmental pollution and it affects the quality of environment and natural resources. The rate of the environmental pollution depends on an industrial specialization of the territory, the technologies used; another important factor is the environmental monitoring of the activities of economic entities provided by the government and the society [5; 8; 13]. There is a number of principles governing the interconnection between of economical, environmental and social systems:

- the interrelation between the environmental expenditures and the state of the environment. Economists believe that to maintain a stable environmental situation it is necessary to allocate 2-3% of the GDP annually to environmental management, and 5-6% of the GDP if environmental situation is unstable - therefore, there is a interrelation between the environmental quality and economic damage. Researches showed that at unstable environmental situation, even when the impact of anthropogenous factor impact to the natural systems is not increasing, it can account for damage increase rate from 3 to 5 % in year.

- interrelation between the degree of environmental pollution and the level of population morbidity and well-being, as well as its depopulation.

Traditionally to assess the economical development of the region one uses the level of production output and consumption of goods and the increase of these values per capita (gross revenue, gross product, real gross product per capita, the increase rates of these measurements). The economic indicators on their own cannot fully characterize a socio-economic development of the region. The main aspects of human-environmental interaction. During the centuries of mankind evolution the pressure on the biosphere has been growing steadily due to its productive activities. Even at the dawn of the civilization the increase of the scale of human activity resulted in irreversible changes in the environment. So, in China a huge amount of land devoted to rice production entirely changed natural landscapes, removed fauna and flora which had existed earlier and resulted in new climatic conditions on big territories.

In the XX c. in early 30-s a famous Russian scholar V. N. Vernadskij introduced a special term «noosphere» to designate the inhabitancy of the people community altered under their necessity, and substantially distinguished from physical conditions. As a matter of fact it is possible to attribute all big changes in natural atmosphere caused by industrial activity of the mankind to the noosphere: megapolis, agriculture, production of natural resources, technical influence on biosphere with the development of new territories. Resource ecosystem supports the functioning of the biosphere and civilization on our planet. All the resources of the ecosystem can be conventionally separated into renewable and non-renewable. The distinction between lies in the regeneration time: if for the first group of resources it is relatively insignificant (a few decades), then for the second group it exceeds 1000 years. The human activity has already gotten into planetary scale and it requires the creation of a new class of models of development for the states to control their economical activity to preserve the conditions of life on our planet, and consequently, ensure successful development of global economy. We will indicate the main aspects of the problem [1; 2; 3; 5; 6; 7; 11; 12].

1. Energy aspect. In the last decades besides the growth of productivity, technological advances and the improvement of workers' professional skills the industry started to consume increasingly more energy. In the last centuries it was justified, as there was a transition from an ineffective system of production to a highly productive system. It was the energy that contributed the most to the rise of productivity, as it grew at the fastest rate. It is particularly visible in the case of agriculture; in the middle XX c. the average yield of grain in the developed countries increased threefold when the energy costs growth rate of the production of one ton of the grain had increased by two orders. However recently the new forms of activity and technologies appeared that require considerably smaller energy expenditures: microelectronics, biotechnology, robotics. The advancement of modern technologies predominantly happen in the sphere of reduction of energy costs. This tendency predominately manifests itself in the fact that all energy-intensive industries moved from economically developed to

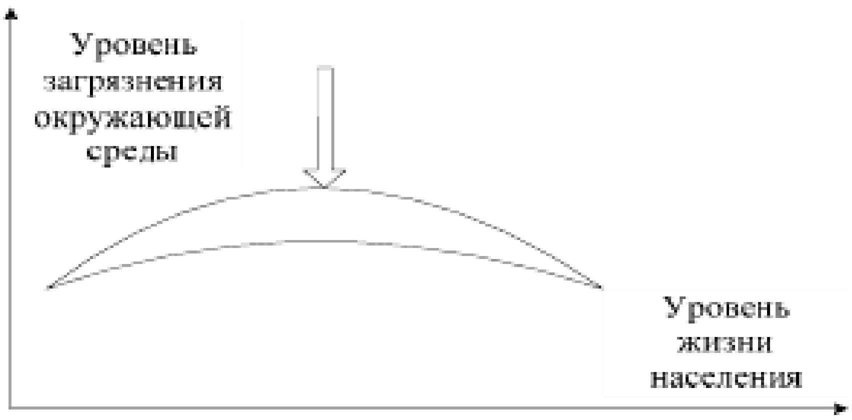
developing countries, in other words, energy absorption capacity becomes one of the most important feature of production.

2. The problem of self-contained technologies. Human industrial activity can be considered as a global technological process of supplying the community with all the necessities. In the last centuries this technology became essentially open: it cannot exist without the non-renewable resources from the Earth depths. There is not only depletion of natural resources, but also shortage of renewable resources, for example, fresh water. That is why in all countries with developed economy significant efforts are made to create and use self-contained technologies that use renewable resources.

3. The problem of pollution. There is a tendency to move not only energy-consuming technologies, but also factories that strongly contaminate biosphere from the developed countries. They include, for example, the extraction of non-renewable natural resources, ferrous metallurgy, some chemical production industries. The quick growth of export of raw materials and metal from less developed countries is connected with it. Besides there is a tendency to move the waste from high-tech factories (for example, radio-active wastes) from the developed countries to the less developed countries. However this process appears dangerous because, above all, it increases the gap between the developed and the less developed countries not only in technological area, but also in the standard of living of the population and the increase of global danger of biosphere destruction.

4. Organizational aspect. The development of the technologies and scientific and technical progress require a continuous improvement of organizational structures of production activity, including on an international level. These problems affect the human-biosphere interaction globally. That's why one has to take into account social and environmental factors and long-term consequences of the decisions made. The models describing these processes have not yet been created.

Industrial process and human life are connected with the production of harmful wastes that get into the environment. Because of it some properties of environment change, and that, on the one hand, causes the decrease of the living standard, and on the other hand - diminishes productive capacities. The possibility of environmentally safe development of the society is based on the man-nature relationship and its patterns. By affecting the condition of the environment with the results of its activity and exploiting territory's natural resources the society creates its health standard both qualitatively and quantitatively, the level of supplying each member with basic nutrition, the level of its depopulation, i.e. all that forms a living standard of the society on a given territory. In turn, population living standard greatly determines the environmental situation. There is a correlation between the quality (level) of living of the society and the change of the environmental pollution which can be graphically described with the Kuznets curve (fig. 1). The Kuznets curve shows that only at sufficiently high level of the population well-being the society will be able to allocate sufficient resources to environment protection activities. At higher level of economical development the given curve (arch) will be more flat.



Picture 1: Kuznets Curve

Adherents of this theory assert that the society in which poverty and injustice thrive will always be subject to environmental and other crises. In that way environmental pressure on the territory depends on the phenomena and processes. Industrial production has greater influence on the environment, than service industry; usage of modern means, the technologies of production reduces the volume of wastes, emissions of pollutants to the atmosphere; the high standard of living of the population corresponds to the higher level of environmental saving. Based on the above one can assume that discrepancies in the levels of economical and social development entail unequal environmental pressure. At the same time high level of economical development doesn't corresponded to a high level of environmental pressure. This fact proves Kuznets theory: the economical development leads to the increase in the standard and quality of living of the population and that combination shapes the environmentalization of the society, the understanding of the necessity of environmental saving. One should consider the fact that the condition of an ecosystem depends on the peculiarities of the region where it is located as one of the characteristics of ecosystems. Environmentally regions can be divided into two megaclusters: the first megacluster consists of the regions with low level of environmental pressure; the second megacluster consists of the regions with high level of pressure («high», «higher than average», «average» levels). Therefore, doing the research and modelling of environemntal systems one should take into account the environmental level of the region [4; 5; 6; 7; 12; 13].

2.2. MATHEMATICAL MODELS OF ENVIRONMENTAL-ECONOMIC SYSTEMS

In the today's world the specialist from the different fields of knowledge constantly have to face the challenges created by the complexity of the world itself both natural (nature) and man-created (technosphere) causes [4; 7; 11]. To solve this problem successfully it's not enough to consider individual elements or particular questions separately. They have to be considered as a whole, in a system, taking into account many interconnections and specific traits. Mathematical modelling of environmental systems is aimed at lowering the environmental pressure of harmful impacts of man's activity, maintaining the natural resources on a highly productive level to use them for indefinite periods of time. The condition of the ecosystem of the region, country, countries is one of the most important factors affecting quality of the human life. The main, final and certain result of the research of environmental systems is a mathematical modelling enabling formal simplification of difficult ecosystems, as an abstract image of the real world. At present time the urgency to take preventive measures to protect specific species and biological communities grows. The vital necessity of environmental forecasts for future economic plans and projects of new technologies related to the usage of natural resources is evident as well. We will call «environmental modelling» such generalizations and methods on which effective management decisions regarding the biosphere can be based on [4; 5; 6; 7; 8; 9; 12; 13; 14].

An ecosystem – biosphere – does not allow experiments on itself in conventional, natural-science sense (laboratory or full-scale experiments; see. Flejshman, 1986; Rosenberg, Brain, 1992); the only way to study it and predict its feature's behaviour is mathematical modelling. And different authors focus on different aspects of biosphere research (environmental, environmental-economic, economical, demographical) and build the models of different types (imitation, analytical, self-organised), offer different ways to achieve the sustainable state of the biosphere (theories of «zero growth», «limited growth», «capital investments redistribution», «growth allocated evenly between regions», «transition to new resources» and so forth). The researchers also make predictions varying from socially-ecological pessimism (J. Forrester, D. Medows) and cautious optimism (M. Mesarovich, E. Pestel, V A. Gelovsni, N. N. Moisseev etc.) to overoptimism (G. Kan, B. Brown). Academician V. I. Vernadskij wrote: «Living biosphere organism should be now empirically studied as a unique body that cannot be reduced to known physical-chemical systems». Biosphere is a unique ecosystem, and its patterns of structure and behaviour have been previously studied only retrospectively. As a whole it is impossible to experiment with biosphere, as challenges of a global experiment are not only of technical nature: the researcher has no right to take chances, changing in one way or another the conditions of existence of such a unique object as the biosphere. So, computer imitation, pseudoexperiment with the model of the biosphere, becomes the only research method of complex system specifications of the biosphere in different scenarios of mankind development [11; 12].

2.3. THEORETICAL ASPECTS OF MATHEMATICAL MODELLING OF ECOSYSTEMS

Let's consider the main theoretical foundations and mathematical models of interindustry analysis to construct mathematical models enabling to describe and to analyze processes in ecosystems on the macro- and micro-levels. The ecosystem of any country, region is a complex dynamic system, with a big variety of inside and outside connections. A complex system is system that has at least one of the following properties: 1) big number of variables and big number of connections between them and other parts of the system; 2) big number of equations describing the system; 3) big number of non-linear elements appearing in the system. The parameters of complex systems are frequently of a non-numerical nature leading to the dependency of their qualitative analysis and synthesis from topology and metrics introduced in the measured environment. In scientific literature there exist such complex systems as multidimensional, multiconnected, multicomponent, highly complex, big, multilevel [4; 7; 12]. We will mention briefly the following kinds of the systems characterized by different degrees of complexity:

1. Conservative. These are systems, in which there is at least one element that doesn't change under the influence on external environmental factors. Such systems are structurally unstable and are characterized by the existence of equilibrium point.

2. Dissipative. These are the systems, in which irreversible processes can exist.

3. System with equilibrium and non-equilibrium limits is a system, in which parameters can be fixed on a constant level or can have a few steady conditions.

4. Systems with linear behaviour are the simplest among the complex systems. However, the presence of feedback introduces an additional complexity in them.

When a system is non-linear, or, alternatively it becomes such in some area, then one of the most important factors of complexity referred to as the ambiguity arises.

5. Isolated systems with inevitable transition into a steady state. The transition of the system into an equilibrium state here is equal to the condition of balance here is equaled to the sustainability of the system.

6. Dynamic systems are systems, with states varying with time. The largest increase of complexity is caused by the system going to a steady non-equilibrium state. The complexity also increases, if processes occurring in the system fluctuate Among all kinds of dynamic systems the most complex ones are discrete dynamic system.

7. Systems with algorithmic representation of information. Here algorithm complexity is the length of the shortest way to record the finite sequence of characters. The most difficult algorithmic representation is the description of a random process [4; 7; 10; 11].

Economical-environmental systems should be classified as a complex system which require a specific approach and a special toolkit.

2.4. STATIC MODEL OF INTERINDUSTRY ANALYSIS

The models of interindustry analysis are applied when it's necessary to calculate volumes of several kinds of output based on the established demand and is used in industrial activity of all branches of the system or to define the final demand based on the established plan of output [3; 5; 7; 8; 9; 11; 13; 14]. The interindustry analysis is based on statistical tables giving the picture of the dynamics of interindustry linkages for determined period of the time. We will consider system of the branches of national economy *n* where each of these branches produces one product and can consume in its activity the output of all branches of the system. Here the question arises about the output of each branch meeting the market demand and the needs of all branches of the system. Let's introduce the notation:

x_i - output volume produced by the branch *i*, $i = \overline{1,n}$;

x_{ij} - output volume of the branch *i*, needed for output production by a branch *j*, $i, j = \overline{1,n}$;

c_i - final demand on the output of a branch *i*, $i = \overline{1,n}$ which detirmines the remaining part of the output of the branch *i* after the output by all the branches of the system has been produced:

$$c_i = x_i - \sum_{j=1}^{n} x_{ij}, \quad i = \overline{1,n};$$

a_{ij} - factors of direct material cost determining the output volume of a branch *i*, needed to produce a unit of output of a branch *j*, i.e.

$$a_{ij} = \frac{x_{ij}}{x_j}.$$

From the last formula we get $x_{ij} = a_{ij} \cdot x_j$. We will substitute x_{ij} into the formula for the final demand and we will get the simultaneous equations:

$$c_i = x_i - \sum_{j=1}^{n} a_{ij} \cdot x_j, \quad i, j = \overline{1,n},$$

or to expand:

$$\begin{cases} c_1 = x_1 - a_{11}x_1 - a_{12}x_2 - \ldots - a_{1n}x_n, \\ c_2 = x_2 - a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ c_n = x_n - a_{n1}x_1 - a_{n2}x_2 - \ldots - a_{nn}x_n. \end{cases}$$

We will rewrite the last simultaneous equations as

$$\begin{cases} (1-a_{11})x_1 - a_{12}x_2 - \ldots - a_{1n}x_n = c_1, \\ -a_{21}x_1 + (1-a_{22})x_2 - \ldots - a_{2n}x_n = c_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -a_{n1}x_1 - a_{n2}x_2 - \ldots + (1-a_{nn})x_n = c_n \end{cases}$$

or in matrix representation:

$$(I - A)x = C,$$

where $A = (a_{ij})$, I is an identity matrix of degree n, $C = (c_j)$, $x = (x_i)$. We will call A the matrix of direct material costs or technological matrix, C - the vector of final demand. We will call the simultaneous equations $(I - A)x = C$ the system of the equations of interindustry balance or balance model by Leontev. We will introduce an important definition.

Definition. Technological matrix A of n degree is called productive, if such non-negative vector of n degree x^0 exists that the following equation is true $Ax^0 < x^0$. The last inequation has a simple economical sense. Indeed, x^0 is a vector (plan) of the output , A - direct costs to issue a unit of output of each kind, then vector Ax^0 represents costs to issue all output, and inequation has such meaning: costs should be less, than the output and that shows the equivalent efficiency of used technology A. Theorem (The conditions of efficiency of technological matrix). The matrix $A = (a_{ij}), (i, j = \overline{1, n})$ is productive, if one of the conditions is fulfilled:

$$1)\ \sum_{i=1}^{n} a_{ij} < 1,\ \ j = \overline{1, n};$$

$$2)\ \sum_{j=1}^{n} a_{ij} < 1, i = \overline{1, n}.$$

Theorem (The condition of nonnegativity of the solutions of the system of the equations of interindustry balance). If technological matrix A is productive, the vector of final demand $C \geq 0$, then the system of the equations of interindustry balance $(I - A)x = C$ has a non-negative solution: $x \geq 0$. Theorem (Hawkins-Smith). For the system of the equations of interindustry balance to have a positive solution at $C \geq 0$ it is necessary and sufficient for all main minor determinants of the matrix $I - A$ to be positive: $d_1 > 0, d_2 > 0, \ldots, d_n > 0$.

2.5. THE OPTIMIZATION MODEL OF INTERINDUSTRY ANALYSIS

Let's suppose that when in the production process resources of the branches that are not part of the considered system are used we will call these resources additional. While creating the optimization models we will use the notation that we have established earlier. Besides let’s assume that

$\mathbf{r} = (r_i), i = \overline{1,m}$ - the reserves of additional resources;

$\mathbf{D} = (d_{ij}), i = \overline{1,m}, j = \overline{1,n}$ - standard matrix of the resources consumption, where d_{ij} is a volume of resource i, needed to issue a unit of production of branch j;

$\mathbf{p} = (p_i), i = \overline{1,n}$ - the vector of prices of the finished goods, where p_i is the price of the unit i of the production;

$\mathbf{w} = (w_i), i = \overline{1,m}$ - the vector of prices for additional resources, where w_i is price a of the unit of resource *i*.

First of all we will consider a simple verbal problem statement. To maximize the price of the final demand at the established prices for the final goods $\mathbf{p}$, resources reserves $\mathbf{r}$ and the standards of their consumption $\mathbf{D}$. To create an appropriate mathematical model we will express the price of the final demand and we will write limitations connected with the reserves of additional resources: (Price of final demand y) = (the sum of the product of the prices and the final demand), (Volumes of resources consumption) ≤(resources reserves),Volumes of output are non-negative or in mathematical notation:

$$y = \sum_{i=1}^{n} p_i c_i = pC,$$

$$Dx \le r,$$

$$x \ge 0.$$

Taking into account that $C = (I - A)x$, we come to following problem of linear programming:

$$y = p(I - A)x \to \max,$$

$$Dx \le r,$$

$$x \ge 0.$$

Such problems are solved by simplex-method. A resource is called scarce, if it is used entirely, otherwise it is called non-scarce. The status (scarce/non-scrace) of a resource can be determined by the value of its balance: if the balance of the resource is positive, then it is non-scarce, if the balance is equal to zero, then it is scarce. Balances of the resources are equal to the values of the appropriate balance variables in the optimal solution. The shadow prices of the resources can be obtained by solving both direct or dual task. We will construct the dual task of the problem. It looks like:

$$s = wr \to \min$$

$$wD \ge p(I - A),$$

$$w \ge 0.$$

The optimal solution of the problem determines the shadow prices of additional resources. We will note that shadow prices of scarce resources are positive, and those of non-scarce ones are equal to zero. We will note that resource with maximum shadow price is the most valuable. It means that a reserve of this resource should be escalated primarily to enable a bigger rise in prices of final demand for a one unit increase of resource reserve.

The suggested model can be used to solve a problem of finding the optimal output that maximizes the income gained from selling the finished goods while keeping with the standard of environmental pollution and established prices of the finished goods. Let there be *m* types of pollution, matrix *D* - the matrix of extreme concentration (maximum allowable concentration) of pollution, the vector ***r*** **-** vector of the limitations representing admissible emissions of pollution of all kinds for the given production. As example we will consider a hypothetic four-branches system with three kinds of pollution: pollution of the atmosphere, pollution of water objects, pollution by production wastes. We will solve the problem taking into account the given initial data: the technological matrix:

$$A = \begin{pmatrix} 0,03 & 0,06 & 0,09 & 0,00 \\ 0,05 & 0,05 & 0,04 & 0,05 \\ 0,07 & 0,01 & 0,02 & 0,04 \\ 0,06 & 0,05 & 0,04 & 0,09 \end{pmatrix}, D =, \text{the vector of prices} \begin{pmatrix} 0.10 & 0.30 & 0.70 & 0.10 \\ 0.20 & 0.60 & 0.40 & 0.80 \\ 0.03 & 0.08 & 0.04 & 0.09 \end{pmatrix} \boldsymbol{p} =$$

(12,15,21,13), vector of the maximum allowable concentration of pollution of each kind ***r*** =.

$$\begin{pmatrix} 197.3 \\ 771.9 \\ 45.8 \end{pmatrix}$$

Then the problem can be represented as follows:

$$y = 8.64x1+12.67x2+18.38x3+10.24x4,$$

$$0.1x1+0.3x2+0.7x3+0.1x4 \; □ \; 197.3,$$

$$0.2x1+0.6x2+0.4x3+0.8x4 \; □ \; 771.9,$$

$$0.03x1+0.08x2+0.04x3+0.09x4 \; □ \; 45.8,$$

$$x1, x2, x3, x4 \; □ \; 0.$$

The problem of linear programming is solved using simplex-method in the packaged software MAPLE. Such results are received:

$$y_{opt} = 13730.726, x_1 = 1421.647, x_2 = 0, x_3 = 78.765, x_4 = 0.$$

The analysis of the solutions indicates that at permanent technologies and environmental standards of achievement of optimum value of objective function it's better not to produce the output of the second and fourth branches, it may be more profitable to buy abroad or to consider a technological update. Another possible variant is shutting down the production if its modernization costs too much. We will consider the problem of optimization of production with simultaneous observance of defined environmental standards at microeconomic level [5; 7; 9]. Let *p* be the price of produced goods, vector ***w= (w1, w2,…wm)*** represents the cost of elimination of all the kinds of pollution *m*. In the case of excession of the relevant standard, rates of these pollution, they are represented by the matrix of intensity. Then the function of the income *P*, received from the output volume ***x*** looks like

$$P = pF(\mathbf{x}) - \mathbf{w},$$

where □ is a penalty vector: □ $_j$ = 0, at *z j? z j *; **120575; j = 1 at z j> z j *; j=1,2,…, m;* ***z*** – a pollution vector; ***z*** * - the vector of maximum allowable pollution.

Suppose that environmental costs are included in payment for exploitation of nature and the variable ***w*** □ is the payment for limit-exceeding environmental pressure. In this case the function *P* can be considered as the production function dependent on resource volumes ***x***, volumes of pollution ***z,*** maximum allowable pollution ***z*** * and payment for the violation of environmental norms ***w.*** In such a way, a problem of maximization of function *P* at non-negative vectors ***x, z*** and the limit $A\mathbf{x}^T$ □ $\mathbf{b}^T$ is defined, where ***A*** is a matrix of factors of direct material inputs coefficients, b – a vector of limitations determined by production capacities. In this case the production capacities, as well as standards of impact of the production on the environment and costs connected with the excess of environmental pressure are taken into account. This model can be used on the micro-economics level for individual company, industrial complex or industry output [7]. The model of interindustry analysis can be used to analyze the case of increase of resources consumption. To reduce technical wastes it is necessary to spend more resources per unit of output which leads to increased coefficient of direct material inputs which is possible for productive technological matrix [5; 7]. The equation of interindustry balance looks like:

$$(I - A)x = C ,$$

where I is an identity matrix,

A is a technological matrix,

$\mathbf{x}$ is an output vector,

C is a vector of final demand.

Let us assume that for environmental and nature protection activities it is necessary to increase an in-industry consumption, then we will have a new technological matrix, new output vector, a new vector of final demand:

$$A1=A+\Delta A,\ \mathbf{x}^*=\mathbf{x}+\Delta \mathbf{x},\ C^*=C+\Delta C.$$

Then we will have a new equation of interindustry balance

$$(I - A1)\ \Delta\mathbf{x}=\Delta A\mathbf{x}+\Delta C.$$

Let us assume that for accuracy all elements of the matrix A are increased in 1+ times, and the value of α is such that matrix $A1$ remains productive. Then $\Delta A=\alpha A$ and the equation can be written as follows:

$$(I - (1+)\ A)\ \Delta\mathbf{x} = (\mathbf{x}-C) + \Delta C.$$

For productive matrix $A1$ there is an inverse matrix $(I - A1)^{-1}$ with non-negative elements, therefore even in the case of zero increment of the vector of final demand the output vector increases $\mathbf{x}$-$\mathbf{y}$> 0. As shown earlier to model an environmental-economic systems in the macroeconomic and in the micro-economics it is possible to use the interindustry analysis. In the models of multi-sectoral economy it is necessary to take into account payments for environmental activities. Then one needs a model of equilibrium prices [5; 7; 13]. The model in matrix form looks like this:

$$\mathbf{p} = A^T\mathbf{p} + \mathbf{w},$$

where p and w are respectively the vector of prices for industry output and the vector of added cost. Resources used to eliminate the pollution, increase the components of the vector of added cost w. Then the vector of prices per industry output at environmental costs will look like:

$$\mathbf{p^* = (I - AT)\text{-}1(w + wec).}$$

So, the vector of output prices will increase by

$$\mathbf{\Delta p = p^* - p = (I - AT)\text{-}1wec.}$$

The matrix A is assumed to be productive, hence, the matrix $(I - A^T)^{-1}$ is non-negative, the vector of environmental pressure $\mathbf{w}_{ec} > \mathbf{0}$, therefore Δp > 0 at permanent technology of output [7]. We will test the suggested model on a hypothetic example, we will measure environmental pressure with environmental activities payment of 20% of added value for the first branch, 10% for the second branch, 15% for the third branch, 10% for the fourth. The calculations can be made in mathematical packaged software MAPLE:

$$\boldsymbol{W} = (I\text{-}A)\,\boldsymbol{p} = \begin{pmatrix} 8.85 \\ 12.16 \\ 19.07 \\ 9.57 \end{pmatrix},\ \boldsymbol{W}_{ec} = \begin{pmatrix} 1.770 \\ 1.216 \\ 2.860 \\ 0.957 \end{pmatrix}.$$

Let's calculate the increment of price vector which compensates expenditures to decrease of the level of pollution:

$$\Delta\boldsymbol{p} = (I\text{-}A)^{-1}\boldsymbol{W}_{ec} = \begin{pmatrix} 2.216 \\ 1.604 \\ 3.151 \\ 1.418 \end{pmatrix}.$$

In such a manner, at the permanent technology of output to minimize the pollution it is necessary to rise prices for the goods of each production of each branch of industry. Suggested models can be implemented in any mathematical packaged software and let schedule an effective production management strategy aimed at decreased environmental pressure to make optimal managerial decisions.

3. DISCUSSION

The current state of environmental modelling is characterized by 4 main principles (Rozenberg, 1984, Brusilovskij, 1987); The first of them is multiple models principle; It was suggested by V. V. Nalimov (1971) and it is that to forecast a particular environmental situation it is possible to construct a few, equally reliable mathematical models. The second, very important principle is a principle of omnipotent factors that was offered by V. V. Nalimov (1983) as well. In essence it states that there are omnipotent factors which yesterday and today did not and do not play significant role in the dynamics of one ecosystem or another, but which can have significant influence on it tomorrow. The next principle was suggested by the American J. Forrester (1977; 1978), who was the author of system dynamics - it is the principle of counterintuitive behaviour of complex systems. In accordance with this principle complex environmental systems do not behave in a way our intuition tells, i.e. behave counterintuitively. The reason of such behaviour is objective complexity of ecosystems, subjectivity of our knowledges and principle of ommnipotent factors [12]. The fourth is the principle of discrepancy between the accuracy and the complexity suggested by L. Zade (1974; 1976) and which is stated as follows: the concepts of «accuracy» and «complexity» in forecasting the structure and behaviour of ecosystems are connected by inverse relationship - the deeper a real ecosystem is analyzed, the less determined are our statements about its behaviour. On the one hand, the model constructed should be simple mathematically, so that it could be studied by the toolkit available. On the other hand, as a result of all simplifications it should not lose the problem substance. When modelling environmental-economic systems very extensive coverage of mathematical methods and models is used: recessive-correlation analysis, temporary series, multivariable statistical analysis, the theory of random processes, the theory of mass service, expert evaluations, the theory of fuzzy set, the theory of control, imitation modelling. The choice of the method of research depends on research purposes [4; 7; 11; 12].

4. CONCLUSIONS

The variety of aspects of environmental problems, significant scales of manifestation and complexities of their solutions caused by economical reasons and industry features, require scientifically based approaches aimed at the environmental improvement in the conditions of the scarceness of available resources. The implementation of requirements for the sustainable development of modern economical system from small enterprises and various branches to regional economies, involves efficient management of any economical system which is connected with the need for evaluation and maintenance of mechanisms of its security. In this light ensuring the sustainable development of industrial enterprises involves maintenance of their environmental-economic security characterized by the ability of the system of the enterprise not to change its internal communications and specifications of functioning under the influence of internal and external factors in sufficiently long period of time. In this case economical component of environmental problems is emphasized. As a method of research the theory of interindustry analysis is chosen enabling the solution of economical problems connected with environmental pressure by industrial facilities with the help of optimization models. Optimization models of interindustry analysis have as their advantages the mathematical simplicity and a toolkit for implementation of these models. Notwithstanding the simplicity of the suggested models the results of their implementation are quite realistic and allow to make efficient decisions lowering environmental pressure of operating enterprises.

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